# Legendre Polynomial Interpolation of a Time-Dependent Signal

Given the time-dependent signal , it is assumed that the signal can be expressed as a superposition of interpolation polynomials:



where  is an *n-th* order polynomial defined over a range  , and  are constant coefficients. The constant coefficients  can be computed by computing the set of moments:



for . This leads to a linear system of equations that can be used to solve for the . If the  are orthogonal polynomials, such as a Legendre polynomials, this leads to a diagonal linear system of equations, and can be directly used to compute . This is done in the following.

First, introduce the mapping, that will map the range , which is the range of Legendre polynomials. To this end, introduce



It is easily shown that  and . Next, let:

,

where  is the n*-th* order Legendre polynomial. Next, from the right-hand side of ,



Since,

,

then . Therefore, can be expressed as:

,

and  is the Kronecker delta-function. This latter relationship is due to the orthogonality of the Legendre polynomials over the domain . Therefore, from and



Or,



Next, assume that  is uniformly sampled with time step , such that  and . We can also define . Consequently, the integration in can be computed using a trapezoidal rule as:



Then, combining and ,

,

where, then from



The concept of applying this method is that the bounds of  will for a sliding window roughly centered over the value of  that is wanted to be estimated. By limiting *p* to orders 2 or 3, it is hoped that this method will smooth out a significant amount of high frequency noise.

The derivative of the signal can then be estimated having smoothed out the data:



where  was computed in , and  is the derivative with respect to the argument.

As an example, Figure 1 illustrates the interpolated signal for a Hall-probe signal, with 40,500 samples and . A window of 500 samples was used for  (that is, ). The interpolated signal was computed every 50 samples. Namely, the 40,500 sampled signal was effectively sampled more coarsely with 810 samples. The order of the Legendre polynomial expansion was chosen to be *p=2.* Figure 2 shows a zoomed in view of the interpolated signal showing the relative smoothness.

The derivative of the signal is illustrated in Figure 3. While this is still relatively noise, it is much improved compared to the filtering methods.

Finally, Figure 4 repeats this experiment with a 1000 point sample and computed every 100 samples. Additional experimentation is still needed.



Figure 1. Hall probe voltage using a Legendre Polynomial interpolation with *p=2*, a 500 sample window, and sub-sampled every 50 time steps.



Figure 2. Zoomed in view of Figure 1.



Figure 3. Derivative of the Hall probe voltage using a Legendre Polynomial interpolation, with *p=2*, a 500 sample window, and sub-sampled every 50 time steps.

 

Figure 4. Hall probe voltage and its derivative using the Legendre Polynomial interpolated, *p=2*, a 500 sample window, and sub-sampled every 50 time steps.